

Sept. 20

Transition Matrices

Business and political analysts rely on the assumption that a person's decision can be predicted based on prior behavior and statistical knowledge. Usually probability is the most common statistical tool used to make these predictions.

Example:

Terry and Bob are playing chess. They each have an equal chance of winning the first game. If Terry wins his confidence increases and his chance of winning the next game increases to 70%. If he loses then his chance of winning the next game decreases to 45%.

Initial probability matrix $P = \begin{bmatrix} 0.50 & 0.50 \end{bmatrix}$

To calculate the probabilities for subsequent games, we need to create a **transition matrix**. This is a **square matrix** based on the **information in the problem**

$$T = \begin{matrix} & \begin{matrix} T & B \end{matrix} \\ \begin{matrix} T \\ B \end{matrix} & \begin{bmatrix} 0.70 & 0.30 \\ 0.45 & 0.55 \end{bmatrix} \end{matrix} \rightarrow \begin{matrix} \text{All rows} \\ \text{must total} \\ 100\% \end{matrix}$$

To find the probability P_n after “n” rounds of an experiment we multiply the initial matrix by the transition matrix raised to the power of “n”

$$P_n = P \times T^n$$

a) Using the above information determine the probability that Terry will win the second game.

$$P \times T = \begin{bmatrix} 0.575 & 0.425 \end{bmatrix}$$

Terry has a 57.5% chance of winning the next game.

b) Determine the probabilities after the 6th game.

$$P \times T^6 = \begin{bmatrix} 0.5999 & 0.4002 \end{bmatrix}$$

Example #2:

As of January 1 2001 there were 2.8 million people living in Alberta and 900,000 people living in Saskatchewan. Each year, 5% of the people in Alberta move to Saskatchewan and 27% of the people in Saskatchewan move to Alberta.

- 1. Write an initial ~~probability~~ matrix.

$$P = \begin{matrix} & \begin{matrix} AB & SK \end{matrix} \\ \begin{matrix} 2,800,000 & 900,000 \end{matrix} \end{matrix}$$

- 2. Write the transition matrix that represents the shift in population.

$$T = \begin{matrix} & \begin{matrix} AB & SK \end{matrix} \\ \begin{matrix} 0.95 & 0.05 \\ 0.27 & 0.73 \end{matrix} \end{matrix}$$

From SK

- 3. Based on these predictions find the population in Alberta and Saskatchewan

- a) January 2002

$$[A] \times [B]^1 = \begin{bmatrix} 2,903,000 & 797,000 \end{bmatrix}$$

- b) January 2003

$$[A] \times [B]^2 = \begin{bmatrix} 2,973,140 & 726,960 \end{bmatrix}$$

Telus: 14

Rogers: 2

$$P = \begin{array}{c} T \quad R \\ \left[\begin{array}{cc} 14 & 2 \end{array} \right] \end{array}$$

36% of Telus users switch to Rogers
and 7% of Rogers users switch to Telus.

$$T = \begin{array}{c} T \quad R \\ \left[\begin{array}{cc} 0.64 & 0.36 \\ 0.07 & 0.93 \end{array} \right] \end{array}$$

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